Science Olympiad
Science Olympiad National Tournament 2024
May 25, 2024

Reach for the Stars B Answer Key

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Question 1 (40 points)

For each question, give 1 pt for the correct constellation and 1 pt for the correct image(s).

a. [2 pts] None, Image 8, Image 11

b. [2 pts] B, None

c. [2 pts] L, None

d. [2 pts] E, None

e. [2 pts] None, Image 4, Image 17

f. [2 pts] M, Image 7, Image 21

g. [2 pts] None, Image 2

h. [2 pts] D, Image 18

i. [2 pts] H, Image 16

j. [2 pts] F, None

k. [2 pts] N, None

l. [2 pts] G, Image 3

m. [2 pts] F, Image 13, Image 14

n. [2 pts] I, Image 6

o. [2 pts] H, Image 1

p. [2 pts] A, Image 10, Image 20

q. [2 pts] C, Image 15

r. [2 pts] J, Image 19

s. [2 pts] H, Image 1

t. [2 pts] K, None

Question 2 (20 points)

a. [1 pt] Star-forming region OR HII region

b. [1 pt] L, J, G

c. [1 pt] Protoplanet

d. [1 pt] Rapid star formation

e. [1 pt] Protostar in the center and outflow cavities on both sides

f. [1 pt] Light emitted primarily from gas clouds

g. [1 pt] Radio

h. [1 pt] An outflow jet and a Bok globule

i. [1 pt] Sun may have had similar outbursts, producing radioisotopes

j. [1 pt] It is being sculpted out by forming planets

k. [1 pt] Young stellar objects

l. [1 pt] Most distant starburst galaxy at time of discovery

m. [1 pt] Unrelated background galaxy

n. [1 pt] Faint brown dwarfs

o. [1 pt] Image 22

p. [1 pt] Protoplanetary disk

q. [1 pt] Formation may occur in filaments rather than puffs/clouds

r. [1 pt] Images 25 and 28 respectively

s. [1 pt] Protostar on the left and variable nebula on the right

t. [1 pt] The Jeans Criterion

Question 3 (18 points)

a. [1 pt] Different molecules in the cloud.

b. [1 pt] All organic/contain carbon.

c. [2 pts] Most abundant species is methane/CH_4 (1pt); found at ∼7.7μm (1pt).

d. [1 pt] Infrared.

e. [1 pt] Able to observe through dust of collapsing cloud.

f. [2 pts] Would see a decrease in amplitude for chemical species given, shift to shorter wavelengths (1pt); hydrogen would dominate (1pt).

g. [3 pts] As heat and pressure in the collapsing cloud increase (1pt), dust grains or particles serve as the substrate onto which smaller chemical species bond to one another (1pt). This can only occur when the cloud has collapsed enough to the point that protostars are forming, producing sufficient heat/pressure (1pt).

h. [4 pts] The likely structures of this system would be:

   (i) Protostar with rocky planets forming (not long enough to begin fusing hydrogen, heavy species will build planets)

   (ii) Main-sequence star with planetary system (∼enough time to fuse hydrogen, planets will have accreted)

i. [3 pts] Identification:

   (i) NGC 1333

   (ii) James Webb Space Telescope

   (iii) Younger

Question 4 (24 points)

a. [1 pt] *Chani* appeared brighter on average.

b. [2 pts] *Atreides* is most likely an FU Orionis variable (1pt), while *Chani* is most likely a T Tauri variable (1pt).

c. [2 pts] *Chani* is the closer star to Earth.

d. [1 pt] No temperature change to the protostar occurred over the course of brightening.

e. [4 pts] *Atreides*: -4.0 (2pt); *Chani*: -1.0 (2pt)

f. [4 pts] *Atreides*: 2.5^4 times brighter (2pt); *Chani*: 2.5^1 times brighter (2pt).

g. [2 pts] It corresponds to a contraction of the protostar.

h. [4 pts] *Atreides* saw the greater contraction (2pt), by a factor of 2.5^3 times (2pt).

i. [4 pts] Potential answers (1pt each) include: isothermal contraction is unrealistic; brightness changes aren't necessarily intrinsic; contraction in a quick spike as in *Chani* is unrealistic; a single transient event like in *Chani* could be due to data error or malfunction; any other logical answer (variable).

Question 5 (18 points)

a. [1 pt] Barnard 68

b. [2 pts] 30000 – 50000 arcsec^2

c. [2 pts] $C^{18}O$

d. [3 pts] RA: 60 – 80 arcsec; Dec: -80 – -100 arcsec

e. [3 pts] Photons from behind or with the collapsing cloud impact the molecule (1pt), which creates a rotational moment of inertia with as much energy as that photon imparted (1pt). The cloud then re-emits a photon at the energy detected per plot, dissipating its rotation (1pt).

f. [3 pts] The optical image shows significantly shorter-wavelength observations than the other three (1pt). These density plots, being focused on individual molecules, represent an image of the region at just one wavelength, the one that that molecule absorbs/emits (1pt). These are all at different, longer wavelengths than optical images, producing a different appearance (1pt).

g. [4 pts] The density pockets exert a gravitational force on their surroundings (1pt), which is balanced by outward thermal pressure (1pt) generated within the cloud as the dense regions rub against each other or absorb/re-emit energy (1pt). Thus, a thermal-force map of the same region would look like an overlap of all densities in the cloud, with higher outward pressure force in greater areas of density (1pt).

Question 6 (30 points)

a. [3 pts] **Star 2** is more massive since it is closer to the center of mass. **Star 1** has a larger velocity.

In the question, we are treating the two stars as a closed system where the center of mass does not move. So, the total momentum of the system is 0, and from conservation of momentum, it always has to be 0. Writing this out, we get:

$$m_1 v_1 = m_2 v_2$$

where m_1 and m_2 are the masses of Stars 1 and 2, and v_1 and v_2 are the speeds of Stars 1 and 2. (In principle, there could also be a negative sign to take the direction of the stars' motion into account, but this negative would eventually be multiplied by another negative, which is why I didn't include it.) From earlier in the part, we determined that $m_2 > m_1$. So, in order for the two sides of the above equation to be equal, v_1 must be greater than v_2.

For more information on this, check out this great document written by Daniel Fleisch and Julia Kregenow for their textbook, *A Student's Guide to the Mathematics of Astronomy*. It is one of best-written textbooks I have ever seen in my life!

b. [3 pts] Your friend should use $a_1 + a_2$, the separation between the two stars. The two stars need to have the same period around the center of mass so they always stay on "opposite" sides of their orbits relative to each other. Otherwise, the center of mass would start to move, which cannot happen since the total momentum of the system has to be 0.

If you're curious about the underlying math, check out this thread on Physics Stack Exchange or this page from NASA.

c. [2 pts] In general, for a system of two objects of masses m_1 and m_2 separated by a distance r, the force of gravity between the two objects will be $F_{\text{gravity}} = Gm_1m_2/r^2$. So, for the "real" system in Fig. 5(a):

$$F_{\text{gravity, real}} = \frac{Gm_1m_2}{(a_1 + a_2)^2}$$

For the "reduced-mass" system in Fig. 5(b), where $a \equiv a_1 + a_2$:

$$F_{\text{gravity, reduced}} = \frac{G(m_1 + m_2)\mu}{a^2} = \frac{G\cancel{(m_1 + m_2)}\frac{m_1 m_2}{\cancel{m_1 + m_2}}}{(a_1 + a_2)^2} = \frac{Gm_1m_2}{(a_1 + a_2)^2}$$

Both of these are the same! "Reduced-mass" systems are useful because they simplify the motion while keeping things like force and angular momentum the same as the "real" system.

d. [4 pts] We begin by equating the centripetal force with the force of gravity we calculated in the previous part, just as the hint tells us:

$$\frac{\mu v^2}{a} = \frac{G(m_1 + m_2)\mu}{a^2}$$

Solving for v gives our answer:

$$v = \sqrt{\frac{G(m_1 + m_2)}{a}}$$

e. [5 pts] The kinetic energy of the system is the sum of the kinetic energies of both of these objects:

$$\text{KE} = \overbrace{\frac{1}{2}(m_1 + m_2)v_{m_1+m_2}^2}^{\text{KE of object at center}} + \overbrace{\frac{1}{2}\mu v_\mu^2}^{\substack{\text{KE of} \\ \text{object} \\ \text{of mass } \mu}}$$

In the "reduced-mass" system, the central object is stationary, while the object of mass μ moves around it. So, the first term is 0, since the velocity of the central object is 0. The total kinetic energy of the system is then:

$$\text{KE} = \frac{1}{2}\mu v_\mu^2 = \frac{1}{2} \times \frac{m_1 m_2}{\cancel{m_1 + m_2}} \times \frac{G\cancel{(m_1 + m_2)}}{a} = \frac{G m_1 m_2}{2a}$$

The gravitational potential energy is a little easier:

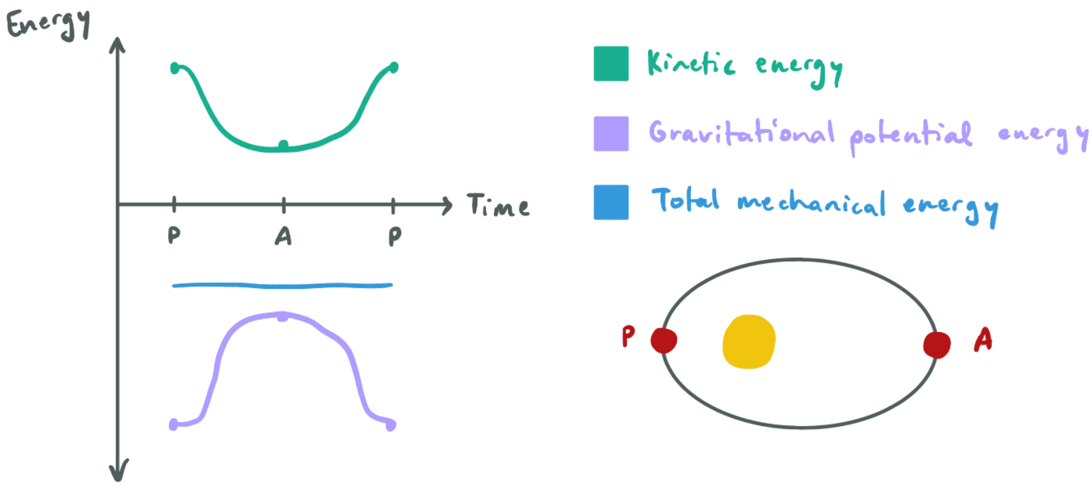
$$\text{GPE} = -\frac{G m_1 m_2}{a}$$

Adding these together, we get that the total mechanical energy is:

$$\text{KE} + \text{GPE} = -\frac{G m_1 m_2}{2a}$$

G, m_1, m_2, and a are all positive, so it follows that the total mechanical energy is negative. In general, if an orbit is bound (i.e., circular or elliptical), then the total mechanical energy is negative. Orbits with positive values for the total mechanical energy are hyperbolas, while those with zero energy are parabolas.

f. [6 pts] The sketch of the graph should look approximately like so:



The exact shape and scaling isn't important, especially considering that this was only a sketch. Some key features:

1. Kinetic energy (KE) is most positive and gravitational potential energy (GPE) is most negative at periastron (P).
2. KE is least positive and GPE is least negative at apastron (A).
3. The total mechanical energy (KE + GPE) stays a negative constant throughout the orbit.
4. Since the reduced mass travels faster at periastron than apastron, the "spike" at periastron is thinner than that at apastron.

Notably, the ratio of kinetic energy to potential energy in the system changes — that is, the virial theorem does **not** hold at every point in the orbit! The virial theorem applies at every point in a circular orbit, but in this elliptical orbit, at periastron, the KE makes up a larger fraction of the total energy, while at apastron, it makes up a smaller fraction. The virial theorem is only valid when we average with respect to time over the entire (or many) orbit(s).

g. [3 pts] When the two stars come closer together, their gravitational potential energy becomes more negative. Since energy is conserved, the more negative gravitational potential energy has to be compensated by more positive kinetic energy. Some of the stars that gained kinetic energy started moving so quickly that they were no longer gravitationally bound to the system and got ejected.

h. [2 pts] One of the assumptions underlying the virial theorem is that the positions and velocities of the objects in the system are bounded for all time. When the stars got ejected from the system, this assumption was violated. In principle, if the stars are ejected from the system slowly, then the virial theorem will be approximately true during the time in between ejections. This time should be long compared to the time it takes for the stars to orbit each other. Students got points regardless of whether they said yes or no as long as they understood the assumptions and limitations of the virial theorem.

i. [2 pts] The second law is a direct result of conservation of angular momentum. So, it would still be valid, since angular momentum is conserved in systems with a central potential. The first and third laws are consequences of gravity scaling with $1/r^2$ and would not be valid if gravity scaled with $1/r^3$.

   For more information on why angular momentum is conserved in systems with central forces, check out Section 2.2 in this document by David Tong or this thread on Math Stack Exchange.

Question 7 (40 points)

a. [4 pts] Example goal: Perform observations of high-energy astrophysical phenomena in dense interstellar environments.

b. [8 pts] Example science requirements:

   (i) The mission shall be able to detect high-energy astrophysical events in the x-ray regime

   (ii) The mission shall be able to detect both quiescent and rapid transient x-ray events

   (iii) The mission shall be able to resolve individual x-ray events in star-forming regions

   (iv) The mission shall be able to observe entire star-forming regions in their totality at once

c. [8 pts] Example engineering requirements:

   (i) The mission shall conduct observations between photon energies of 0.1 keV and 10 keV

   (ii) The mission shall conduct observations of no less than 5 seconds per exposure

   (iii) The mission shall have an exposure resolution of no less than 0.01 arcseconds

   (iv) The mission shall have a field of regard of no less than 3 arcseconds by 3 arcseconds

d. [14 pts] For every requirement addressed, competitors receive **3 points**, with **2 points** extra applied for completion

e. [6 pts] Free space – anything creative, fun, expressive, or unique counts for full credit fits here!