Science Olympiad
Astronomy C
National Event - Michigan State University

May 25, 2024

Answer Key

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Section A (52 points)

1. (a) Cold Molecular Clouds (dark or absorption nebulas acceptable) (0.5 pts for molecular clouds or bok globules)
   (b) Photoionization (erosion due to high energy radiation) (young stars)
   (c) Image 6
2. (a) Shock waves from X-ray outbursts, stellar wind, or magnetic fields → jets
   (b) Image 1 or 16
3. (a) T Tauri, Herbig Ae/Be (accept Herbig)
   (b) F, Z
   (c) Core fusion (0.5 pts) of hydrogen (0.5 pts) to helium stabilizes
4. (a) NGC 1333, Image 15
   (b) Herbig Haro or HH, HH 7-11
   (c) Ionized jets of gas from the newly forming protostar collide with the clouds of gas and dust within the disk material
5. (a) Luhman 16 A/B (accept Luhman 16), Image 4
   (b) [1.5 pts] Luhman 16A has banded/striped clouds, Luhman 16B has patchy clouds, Image 11
   (c) R, M
   (d) 2M1207, Image 10
6. (a) TW Hya, Image 18
   (b) Image 22
   (c) Carbon monoxide (do not accept methanol as CO is required to form it), Image 2
7. (a) Disk or gravitational instability – massive disk breaks into clumps which form protoplanets
   (b) AB Aur, Image 9
   (c) Image 23
   (d) HD 169142, Image 19
8. (a) [1.5 pts] Carbon dioxide & sulfur dioxide, Image 26
   (b) Image 21, emission spectroscopy, phase curves, or secondary eclipse (accept any)
9. (a) Larger planets block more light $(R_\mathrm{p}/R_\mathrm{s})^2$
   (b) HR 8799, Image 3
   (c) Beta Pictoris, Image 12
   (d) [1.5 pts] Main Sequence, H, B
10. (a) Image 24
    (b) Flares (also accept starspots, rotation, or stellar activity)
11. (a) Debris disks are mostly dust, and protoplanetary disks are mostly gas (also accept optically thin vs. thick, respectively)
    (b) Continuous collisions of small bodies within the disk
    (c) [1.5 pts] Planets/planetesimals, asteroids, comets
12. (a) Trappist-1, P
    (b) Stellar wind (accept radiation, flaring, or activity)

13. (a) [2 pts] A is more luminous because the area under the curve is greater (and has a larger surface area). B is redder because it emits relatively less blue light than A. (0.5 pts for each correct answer and justification)
    (b) Visible, visible, (mid) infrared, (mid) infrared (0.5 pts for 2/4)
    (c) [1.5 pts] -0.79 (exact)
    (d) [1.5 pts] K[0–3]V (0.5 pts for each part)
    (e) [1.5 pts] B–V because it captures the most variation in color index/SED "slope", whereas [3.6]–[4.5] doesn't change much. (0.5 pts for justification)
    (f) [1.5 pts] Reddening adds color excess which decreases the predicted temperature. For example, a hot star with reddening would look like a cool star without.

14. (a) O or B stars
    (b) Because O/B stars are short-lived, they end up ionizing the stellar nursery they formed in.
    (c) Reactants: A proton and an electron. Products: An (excited) neutral hydrogen atom and a photon. (0.5 pts for 2/4)
    (d) [1.5 pts] In recombination, the un-ionized hydrogen atom produced may be excited. The excited electron cascades down to the ground state and emit lower-energy photons, with the most dominant being the red H-alpha line.
    (e) [1.5 pts] D, because the rate of growth is highest at beginning and slows over time due to the inverse square law and the increase in recombination rate. (0.5 pts for justification)

Section B (15 points)

15. (a) [1 pt] GJ699

 (b) [1 pt] 2019-06-17

 (c) [1 pt] There is a single bright point source near the center of the JS9 window. Also accept if answer references a 'band'.

 (d) [1.5 pts] Primarily soft X-rays

 (e) [1.5 pts] $\sim 800\,\mathrm{eV}$ (accept answer in 750–900 eV)

 (f) [2 pts] 0.247 nm (accept answer in 0.200–0.275 nm)

 (g) [2 pts] They would not reach the Earth's surface (opacity at that wavelength is $\sim 100\,\%$).

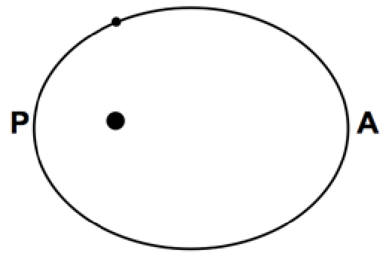
 (h) [1.5 pts] 677155523 seconds (accept answer in 677155250–677155750 seconds or 677155000 seconds)

 (i) [1.5 pts] 11 counts (exact)

 (j) [2 pts] Severe flaring might prevent the star's planets from having a stable atmosphere (atmospheric stripping); life on such a planet would be unlikely.

Section C (33 points)

16. **A Two Body Problem.** In 2000, a planet was discovered orbiting a star in the constellation Orion, with a mass of $1.48\,M_\odot$, a radius of $1\,R_\odot$, and a luminosity of $6.16\,L_\odot$. This particular planet has a mass of $10.4\,M_J$. Its apoastron (A) distance is 5.09 AU and its periastron (P) distance is 2.41 AU. A face-on view of the orbit shown in the image below (not to scale).



The *vis-viva* equation is a key relation that models an orbit of a low-mass, orbiting body around a high-mass, central body:

$$v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right),$$

where

- v is the velocity of the orbiting body (the planet),[1]
- r is the distance between the two bodies,
- a is the semi-major axis of the orbit,
- G is the gravitational constant,
- and M is the mass of the central body (the star).

Solution: For all parts of this question, answers within $\pm 5\,\%$ were accepted.

(a) [1 pt] The conservation of what property leads to the vis-viva equation?

Solution: Conservation of (mechanical) energy.

0.5 points were awarded stating for (consv. of) momentum, kinetic energy, or potential energy.

[1] Where we approximate the central body as "fixed".

(b) [1 pt] Use the vis-viva equation to find the velocity of the planet at apoastron and periastron, in km/s.

> **Solution:** Use the given vis-viva equation to find: $v_A = 12.9\,\text{km/s}$, $v_P = 27.2\,\text{km/s}$.
>
> Answers in m/s were accepted for full credit. 0.5 points were given if the semi-major axis $a = 3.75\,\text{AU}$ was found. Notably, no points were awarded for writing the vis-viva equation as it was provided.
>
> Teams lost points by not converting *all* values to SI base units, not applying the square root, or swapping the values for r and a.

(c) [2 pts] If this system is viewed edge-on from Earth, calculate the range of transit duration, in hours, we **could possibly** observe. Assume that the planet's radius is much smaller than the star's.

(If you did not answer part (b), use $v_A = 30\,\text{km/s}$ and $v_P = 40\,\text{km/s}$.)

> **Solution:** Divide the diameter of the star by the slowest and fastest velocity of the planet to get the transit duration range.
>
> $$t_{\text{transit}} \in [14.2, 30.0]\,\text{h (OR } [9.66, 12.9]\,\text{h)}$$
>
> 1 point was awarded for showing the relationship between distance, time, and velocity/speed.
>
> Teams lost points by forgetting to convert to the correct units or by using the wrong formula (e.g. $\frac{P}{\pi}\sin^{-1}(\frac{R}{a})$ since it does not account for the variation in velocity due to eccentricity).

(d) [2 pts] The Nancy Grace Roman Space Telescope is slated to launch by May 2027. Aboard it is a coronagraph with an inner working angle of 0.15 arcseconds.[2] If this system is viewed face-on, what is the furthest distance from which Roman can still resolve the planet, in parsecs?

> **Solution:** Use the angular diameter equation $\delta = D/d$, where δ is the angle (in arcsec), D is the physical diameter (in AU) and d is the distance (in pc).
>
> $$d_{\text{max}} = 33.9\,\text{pc}$$
>
> 1 point was given for showing the small angle formula/approximation.
>
> Teams lost points by forgetting to convert to the correct units.

[2] Roman Science Sheet, Nov 2021

17. **The Three Body Problem.** In *The Three Body Problem* (2008) by Cixin Liu, Trisolaris is a fictional planet orbiting in a chaotic three-star system. Canonically, Trisolaris orbits in the (non-fictional) Alpha Centauri system. In 2016, the existence of an exoplanet inside the habitable zone of Proxima Centauri was confirmed; however, Proxima Centauri is on such a wide orbit that the planet hardly feels the other two stars in the system. In this problem, we will explore a more interesting three body system.

First, let's do a warmup problem...

(a) [1 pt] Suppose we have an object moving in a central potential, such that its equation of motion $\ddot{\vec{r}} = -\frac{GM}{r^2}\hat{r}$, where $\ddot{\vec{r}}$ is acceleration vector, M is some mass, r is the distance to the origin, and $-\hat{r}$ points towards the origin. If this object's orbit has a semi-major axis of a, what is its period? Give your answer as an expression in terms of M, a, and any relevant constants of nature.

(Hint: This is equivalent to a small planet orbiting around a star of mass M!)

> **Solution:** $T = \sqrt{\frac{4\pi^2 a^3}{GM}}$, from Kepler's 3rd law. Alternatively, $T = \sqrt{\left(\frac{a}{1\,\mathrm{AU}}\right)^3 \left(\frac{M}{1\,\mathrm{M}_\odot}\right)^{-1}}$ yr.
> Note: Although Kepler's 3rd law is the solution to a 2-body system interacting under the gravitational force law, it is actually the solution to any $1/r^2$ attractive force law. This is because the 2-body problem can be reduced to a 1-body problem, where a body of mass $\mu = \frac{m_1 m_2}{m_1 + m_2}$ orbits around a stationary mass $M = m_1 + m_2$. In the limit of a small planet orbiting around a large planet ($m_1 \gg m_2$), $\mu \approx m_2$ and $M \approx m_1$. See e.g. Wikipedia.

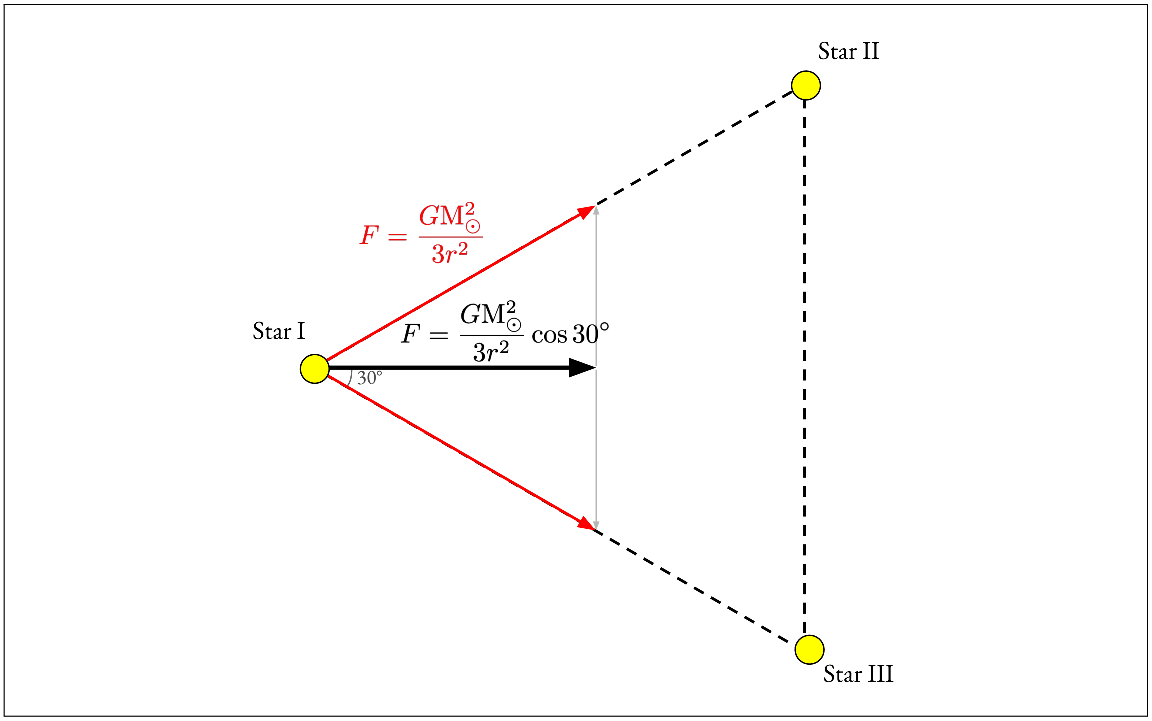
Now, consider a system of three sunlike (i.e. same mass, radius, luminosity, etc.) stars in the stable configuration shown in Image 29, which we will call I, II, and III. Note that each star synchronously follows an identically-shaped orbit; at all points in the orbit, the three stars form an equilateral triangle (gray dashed lines). The black square in the center denotes the barycenter.

(b) [1.5 pts] If the three stars are all a distance r from the center of the system, what is the distance from one star to another?

> **Solution:** This is just finding the side length of the triangle. $2 \cdot r \cos 30^\circ = \sqrt{3}r$

(c) [2 pts] What is the **net** force (magnitude and direction) that Star I feels from Stars II and III? Give your answer as an expression in terms of r, the star mass $M = M_\odot$, and any relevant constants of nature.

> **Solution:** From Newton's universal law of gravitation, the magnitude of the force Star I feels from each star is $F = \frac{GM_\odot^2}{3r^2}$, pointing in the directions of the respective stars. However, the transverse components cancel out. Each star contributes a radial force $\frac{GM_\odot^2}{3r^2}\cos 30^\circ = \frac{\sqrt{3}GM_\odot^2}{6r^2}$. The net force is therefore $F = \frac{\sqrt{3}GM_\odot^2}{3r^2}$, pointing towards the barycenter.



(d) [2 pts] Newton's 2nd law says that Star I will move with equation of motion $M_\odot \ddot{\vec{r}} = \vec{F}$, where $\vec{F}$ is the force you computed in part (c). Which of Kepler's laws apply here, **and why**?

(Hint: Can you equate Star I's equation of motion to that of a planet orbiting a star of some mass?)

Solution: All of Kepler's laws apply here. Kepler's 2nd law is a consequence of the conservation of angular momentum, and thus it always applies. Star I's equation of motion is $\ddot{\vec{r}} = \frac{\sqrt{3}GM_\odot}{r^2}$. Therefore, it moves under an inverse square force law, and its motion is equivalent to that of a 2-body system. Hence, Kepler's 1st and 3rd laws also apply. See note in the part (a) solution.

(e) [2 pts] Each star's orbit has a semi-major axis of 202.4 AU. What is the period of this orbital system in years?

Solution: Star I is moving with the equation of motion $\ddot{\vec{r}} = \frac{\sqrt{3}GM_\odot}{r^2}$. A small planet orbiting a star of mass $\sqrt{3}M_\odot$ would have the same motion. From Kepler's 3rd law, $T = \sqrt{\frac{202.4^3}{\sqrt{3}/3}} = 3789$ years. (Accepted: $[3750, 3800]$ years)

Trisolaris orbits around Star I in a circular orbit with radius 1 AU. Each of Stars I, II, and III have highly eccentric orbits, with a semi-major axis of length 202.4 AU and an eccentricity of 0.997. Currently, **the star system is at apoastron**, and the inhabitants of Trisolaris are enjoying a period of prosperity. Refer to Image 30, where the orbit of Trisolaris is given by the green dashed circle. Because $M_\oplus \ll M_\odot$, you may assume that Trisolaris does not affect the orbits of the three stars.

Trisolaris has Earth-like properties. That is, it has a mass and radius of $1\,\mathrm{M}_\oplus$ and $1\,\mathrm{R}_\oplus$, respectively. Under the ideal greenhouse model, Trisolaris has an albedo of $\alpha = 0.3$ and emissivity $\epsilon = 0.78$.

(f) [1 pt] What is the current distance from one star to another? Give your answer in AU.

> **Solution:** At apoastron, the distance from the barycenter to Star I is $a(1+e) = 404.2\,\mathrm{AU}$. The side length of the equilateral triangle is therefore $404.2\,\mathrm{AU} \times \sqrt{3} = 700\,\mathrm{AU}$.

(g) [1 pt] Compute the flux that Trisolaris currently receives from Star I. Give your answer as a fraction of the solar constant G_{SC}.

> **Solution:** This is exactly Earth's configuration from the sun, so the flux is $1\,G_{SC}$.

(h) [2 pts] Compute the flux that Trisolaris currently receives from Star II. Give your answer as a fraction of the solar constant G_{SC}.

> **Solution:** From the inverse square law,
>
> $$\text{flux} = \frac{L}{4\pi r^2} = \frac{1}{700^2}\frac{L_\odot}{4\pi(1\,\mathrm{AU})^2} = \frac{1}{700^2}\,G_{SC} = 2.0 \times 10^{-6}\,G_{SC}.$$
>
> (Since $1\,\mathrm{AU} \ll 700\,\mathrm{AU}$, we can just average over an orbit and use Star I's distance to Star II for the distance from Trisolaris to Star II. Accepted range: $[1.95, 2.05] \cdot 10^{-6}\,G_{SC}$)

(i) [2 pts] From the combined fluxes of Stars I, II, and III, compute the current equilibrium temperature of Trisolaris in Kelvins, ignoring its atmosphere.

(If you did not answer part (h), use a total flux of $2\,G_{SC}$.)

> **Solution:** The contribution from Stars II and III is negligible. Using the formula
>
> $$T_e = \left[\frac{S_0(1-\alpha)}{4\sigma}\right]^{1/4}$$
>
> and plugging in $S_0 = G_{SC}$, we have $T_e = 255\,\mathrm{K}$. (It is acceptable to write the answer without showing work, since this is the same as Earth's equilibrium temperature. Accepted range: $[250, 260]$ K)
>
> If the dummy value was used, we have $T_e = 303\,\mathrm{K}$. (Accepted range: $[295, 310]$ K)

(j) [2 pts] Calculate the surface temperature in Kelvins of Trisolaris, taking into account its atmosphere. Is Trisolaris in the habitable zone?

(If you did not answer part (h), use a total flux of $2\,G_{SC}$.)

> **Solution:** $T_s = T_e\left[\frac{1}{1-\frac{\epsilon}{2}}\right]^{1/4} = 288.3\,\mathrm{K}$. This is $15\,°\mathrm{C}$, so it is within the habitable zone. (It is acceptable to write the answer without showing work, since this is the same as Earth's equilibrium temperature. Acceptable range: $[280, 300]$ K)
>
> If the dummy value was used, $T_s = 343\,\mathrm{K}$. This is $70\,°\mathrm{C}$, so it is within the habitable zone (less than boiling point). (Acceptable range: $[330, 350]$ K)

Now, let's fast forward to when **the star system is at periastron** (see Image 31). At this point, the stars are so close together that Trisolaris's is now quite chaotic, and its orbit crosses the barycenter.

(k) [1 pt] What is the distance from each star to the barycenter? Give your answer in AU.

> **Solution:** $a(1-e) = 0.607\,\mathrm{AU}$.

(l) [1 pt] Approximate the average **total** flux that Trisolaris receives from its suns **by placing Trisolaris at the barycenter**. Give your answer in terms of G_{SC}.

> **Solution:** At the barycenter, the flux Trisolaris receives from each star is $\frac{G_{SC}}{0.607^2} = 2.71\,G_{SC}$, so $3 \cdot 2.71\,G_{SC} = 8.14\,G_{SC}$ total. (Acceptable range: $[8.10, 8.20]\,G_{SC}$)

(m) [2 pts] Compute the equilibrium temperature of Trisolaris at periastron in Kelvins, ignoring its atmosphere. Is Trisolaris still in the habitable zone?

(If you did not answer part (l), use a total flux of $5\,G_{SC}$.)

> **Solution:** Since $T_e \propto S_0^{1/4}$, $T_e = 255\,\mathrm{K} \cdot 8.14^{1/4} = 430.7\,\mathrm{K}$. This is well above the boiling point, and Trisolaris is no longer in the habitable zone. (Acceptable range: $[425, 435]$ K)
>
> If the dummy value was used, $T_e = 255\,\mathrm{K} \cdot 5^{1/4} = 381\,\mathrm{K}$, also not in the habitable zone. (Acceptable range: $[375, 385]$ K)

Looks like Trisolaris is in trouble in a few millenia...

18. **From Afar.** *(Important Note: Answers to Question 17 are not necessary to answer this question.)*

Now, let's imagine we are on Earth observing the Trisolaran star system, located at a distance of 4.24 light years. Recall from Question 17 that the Trisolaran system is composed of three sunlike stars in the configuration shown in Image 29, and that Trisolaris is an Earthlike planet, orbiting around Star I in a circular orbit of 1 AU. Suppose that from Earth, this system has inclination of 90° (edge-on).

(a) [1 pt] Compute the parallax of the Trisolaran star system in arcseconds.

> **Solution:** $4.24\,\text{ly} = 1.3\,\text{pc}$, and $\frac{1}{1.3\,\text{pc}} = 0.77''$.

(b) [1 pt] Compute the bolometric apparent magnitude of a single star.

> **Solution:** The absolute magnitude of each star is 4.75 (same as the sun). Using the distance modulus formula, $m = M + 5\log d - 5 = 4.75 + 5\log 1.3 - 5 = 0.32$.

(c) [1.5 pts] Compute the *combined* absolute magnitude of the Trisolaran star system.

> **Solution:** The combined luminosity is $3\,\mathrm{L}_\odot$. $M = 4.75 - 2.5\log 3 = 3.56$.

(d) [1.5 pts] Suppose we are looking for exoplanets around this system via the radial velocity method. What is the radial velocity amplitude of Star I caused by the orbit of Trisolaris? Give your answer in m/s.

> **Solution:** The orbital velocity of Trisolaris is $\sqrt{\frac{GM_\odot}{1\,\text{AU}}} = 29784$ m/s. Because Trisolaris and Star I each orbit around their combined center of mass, $m_p r_p = m_\star r_\star$ and hence $m_p v_p = m_\star v_\star$, so the velocity of Star I is $\frac{M_\oplus}{M_\odot} \times 29\,784\,\text{m/s} = 0.089\,\text{m/s}$.

(e) [1.5 pts] If we're looking at the H-alpha spectral line ($\lambda = 656.3\,\text{nm}$) of Star I, what wavelength resolution do we need to detect Trisolaris? That is, compute the corresponding amplitude in wavelength from your answer to part (d). Give your answer in nm.

> **Solution:** $\frac{\Delta\lambda}{\lambda} = \frac{v}{c} = 2.98 \times 10^{-10}$, so $\Delta\lambda = 1.96 \times 10^{-7}\,\text{nm}$.
> This might seem like an extremely tiny number, but the best spectrographs today can (just barely) resolve this shift!