Answer Key:

Finding arc seconds per radian:
(60 arc sec / 1 arc min)(60 arc min / 1 deg)(360 deg / 2π rad) = 206,265 arc sec/rad

Finding the radius of Cas A in radians:
337 pixels (0.5 arc sec/pixel) (1 rad / 206,265 arc sec) = 0.000817 rad

Converting the distance to Cas A to meters:
(11,100 light years)(9.46×10^{15} meters/ 1 light year) = 1.05 X 10^{20} m

Finding the radius of Cas A in meters:
θ = (radius of Cas A) / (distance to Cas A)
radius of Cas A = (θ)(distance to Cas A)
radius of Cas A = (0.000817 rad)(1.05 X 10^{20} m) = 8.58 X 10^{16} m

Finding the mass of gas contained in Cas A:
Density = mass/volume
Volume = 4/3 πr^3
Mass = (density)(volume) = (10^{-21} kg/m^3)(4/3 π)(8.58 X 10^{16} m) ^{3} = 2.65 X 10^{30} kg

Finding the expansion velocity of Cas A*:*
KE = ½ mv^2 = ¼ X 10^{44} J
V = SQRT (2 KE/m) = SQRT [2(¼ X 10^{44} J) / (2.65 X 10^{30} kg)] = 4.35 X 10^6 m/s

Estimating the age of Cas A in seconds:
v = d/t
t = d/v = (8.58 X 10^{16} m) / (4.35 X 10^6 m/s) = 2.65 X 10^{10} s

Converting to years:
(2.65 X 10^{10} s)(1 min / 60 s)(1 h / 60 min)(1 d / 24 h) (1 y / 365.25 d) = 625 y

Displacement of the core remnant:
d = sqrt [$(4292-4286)^2 + (4234-4252)^2$] = 19 pixels
19 pixels (0.5 arc sec/pixel) (1 rad / 206,265 arc sec) = 0.0000460 rad
(0.0000460 rad)(1.05 X 10^{20} m) = 4.83 X 10^{15} m

Average velocity and kinetic energy of the core remnant:
v = d/t = (4.83 X 10^{15} m)/(2.65 X 10^{10} s) = 1.82 x 10^5 m/s
KE = ½ mv^2 = ½ (1.4)(2.0 X 10^{30} kg)(1.82 x 10^5 m/s)^2 = 4.6 X 10^{40} J

NOTE: John Flamsteed observed a supernova in the same location as Cas A approximately 330 years ago. 625 years has the same order of magnitude. With the assumptions and approximations made in this model, this could be the same supernova event, although there are other factors to be considered. Expansion probably does not occur at the same rate in all directions and the rate may not be constant over time.